WAVES IN CHARGED THIN PLATES

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The spectrum of bending waves of a thin conducting plate bearing an electric charge is investigated. It is proved that there exists a critical value of the field upon exceeding which bending instability of the plate occurs.

1. Waves on the surface of a charged conducting fluid have been investigated extensively in recent years (see, e.g., [1]). Turning the electric field on leads to renormalization of surface gravitational waves, and, upon reaching and exceeding the critical field pressure, exponential growth of the amplitude of the waves, known as the Tonks–Frenkel' instability [2], takes place. Electric charges are located within an extremely thin surface layer of the fluid and, since a single charged surface is present, a "negative" field pressure is exerted on the fluid in all cases (even if the surface is not perturbed). It is of interest to investigate the situation where no pressure is exerted on the surface by the electric field in the equilibrium state. It is evident that this situation is realized for a plane conductor layer both surfaces of which bear charges distributed with identical densities. For a fluid, this situation most likely can occur only under conditions of weightlessness and is a subject of experimental and theoretical investigations under microgravity conditions [3]. However, under ordinary terrestrial conditions, this situation is realized in the case of a charged thin plate of an elastic material.

The objective of the present work was to investigate bending waves in thin charged plates and to determine the conditions of violation of their stability.

2. Deformation of thin plates is a traditional subject of elasticity theory [4, p. 54]. When one says that a plate is thin, it is meant that its thickness is small compared to its dimensions along the other two dimensions. The deformations themselves are considered to be small, and the criterion of small deformations is a small displacement of plate points compared to its thickness. Longitudinal and transverse waves can be excited in a thin plate. For an ideal conductor, the spectrum of longitudinal waves remains unchanged upon charging the plate. A different situation occurs for bending waves, for which vibrations occur along the direction perpendicular to the plate plane and, therefore, are accompanied by its bending. The equation of free vibrations of a plate in the absence of a field is written as follows [4, p. 139]:

$$\rho h \frac{\partial^2 \xi}{\partial t^2} + D \Delta^2 \xi = 0 , \tag{1}$$

where $\xi(x, y, t)$ is the deviation of the surface from its equilibrium position along the normal direction, x and y are Cartesian coordinates in the plate plane, and $D = E_G h^3 / 12(1 - \sigma^2)$.

To take the electric field into account, one should insert an expression for the pressure exerted by the field appearing upon deformation of the plate into the right-hand side of Eq. (1) in place of zero.

3. We determine the electrostatic pressure exerted on a charged plate upon its deformation (it is obvious that the electric-force effect is compensated if the plate is plane and nondeformed). The pressure exerted upon the charged surface is directed along the external normal to the surface and equals $E^2/8\pi$. The plate has two charged surfaces, and the difference between the pressures exerted on the surfaces determines the resulting pressure. According to [2], the resulting pressure for a plane surface wave on a plate is as follows:

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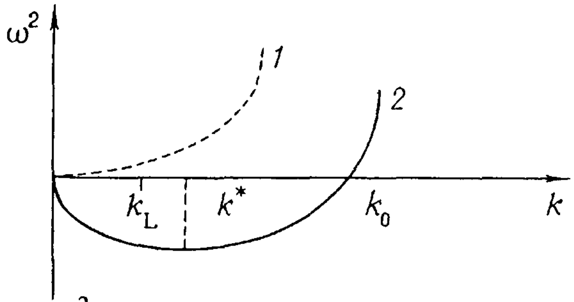

Fig. 1. Dependence $\omega^2(k)$: in the absence of a field (1) and for a charged plate (2).

$$p_e = 2\,\frac{E^2}{4\pi}\,k\xi \tag{2}$$

(the factor 2 appears upon accounting for both charged surfaces). Here E is the electric-field strength in the vicinity of the unperturbed surface. The plane wave for which expression (2) is written is chosen in the following form:

$$\xi = \xi_0 \exp\left(i\,\mathbf{k}\,\mathbf{r} - i\omega t\right), \quad \mathbf{r} = (x, y)\,. \tag{3}$$

4. This choice of formula for the displacement is neither general nor suitable for accounting for arbitrary conditions on the plate boundary (see below); however, it makes it possible to consider free waves on an unbounded plate. Upon substituting (2) into the right-hand side of Eq. (1) and choosing the solution in the form (3), we arrive at the dispersion relationship

$$\omega^2 = \frac{1}{\rho h}\left(Dk^4 - \frac{E^2}{2\pi}\,k\right). \tag{4}$$

A plot of $\omega^2(k)$ is presented in Fig. 1. It is natural to introduce the quantity

$$k_0 = \sqrt[3]{E^2/2\pi D}\,. \tag{5}$$

When the field is turned on, all modes with wave numbers k smaller than k_0 are unstable: their amplitudes grow exponentially with time. Clearly, we are discussing the case of small plate bendings ($\lambda >> h >> \xi$). To describe further changes, one must consider the case of strong bendings ($\lambda >> \xi \geq h$). It is essential that k_0 is a growing function of the field and equals zero at E = 0. In actual situations, due to the finite size of the plate surface, not all waves are realized but only those with $k > k_L$, where $k_L \approx 1/L$ (L being the characteristic dimension of the plate surface). As a result, there exists a critical field value E_L upon surpassing which bending instability of the plate takes place:

$$E_L = \sqrt{2\pi D k_L^3}\,. \tag{6}$$

If the acting value E is smaller than E_L, all bending waves are stable, but their spectrum is deformed according to Eq. (4).

There exists a characteristic value of the wave vector

$$k^* = k_0\,\sqrt[3]{4}\,; \tag{7}$$

if $k_L > k^*$ (and $k_0 > k_L$), the mode with $k = k_L$ has the maximum amplitude growth rate; conversely, if $k_L < k^*$, maximum growth is observed for the mode with $k = k^*$ (it should be pointed out that k^* is found from the condition $\omega'(k^*) = 0$).

5. The considered effect of bending instability takes place for a thin charged plate for which no charge concentration on the edges occurs (only the plate surfaces are charged). This can be achieved if a portion is selected on a plane plate with a large area using, e.g., a dielectric insert clamping somehow a plate with a certain shape. Another possibility exists: a thin plate is placed between two parallel rigid plates having equal charges opposite to those of the plate so as to provide that the total charge of the capacitor with its plates is equal to zero. Exact evaluation of the distribution of amplitudes over the plate surface requires accounting for the boundary conditions along the fixing contour and the corresponding modification of expression (2) to provide coupling between the electrostatic pressure and the displacement. Generally, the boundary conditions for Eq. (1) are rather involved [4, p. 65]; however, in the case of fixed (fastened) plate edges the conditions are as follows [4, p. 66]:

$$\xi = 0 , \quad \frac{\partial \xi}{\partial \mathbf{n}} = 0 , \tag{8}$$

where **n** gives the direction normal to the plate contour (obviously, it is chosen in the plate plane). These conditions mean that if the plate edges are fixed, they cannot experience a vertical displacement, and, in addition, the direction of these edges cannot change.

Modification of expression (2) for an arbitrary bending profile of the plate is complicated by the fact that in general the relationships between p_e and ξ can be approximated as follows:

$$p_e = 2 \frac{E^2}{2\pi} \left(\left. \frac{df}{dz} \right|_{z=0} \right) , \tag{9}$$

where $f(z) = \exp(-Kz)$; this function is a factor in the expression for the change in the field potential near the plate surface, which, due to small deformation amplitudes, is written as follows: $\varphi = E\xi(x, y) f(z)$, and K is a constant (in general a slowly varying function of x and y) and $df/dz|_{z=0} = K$. The expression for p_e is exact in the shortwave limit.

The exact distribution of displacement amplitudes is found upon insertion of (9) into the right-hand side of Eq. (1), solution of this equation with regard for conditions (8), and subsequent refinement of K by the equation $K = -\Delta\xi/\xi$.

NOTATION

ρ, h, E_G, σ, D, density, thickness, Young's modulus, Poisson coefficient, and bending rigidity of the plate; ξ, deviation of the surface from equilibrium; Δ, two-dimensional Laplace operator; E, electric-field strength; **k** and ω, wave vector and frequency of the wave; p_e, pressure.

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